Coherent Crosstalk in Multichannel FSK/DD Lightwave Systems Due to Four-Wave Mixing in Semiconductor Optical Amplifiers

D. J. Blumenthal, *Member, IEEE,* and N. C. Kothari

***Abstract*—Coherent crosstalk in multichannel lightwave transmission systems due to four-wave mixing in semiconductor optical amplifiers is analyzed. For direct detection of evenly spaced frequency channels, coherent beating between signal and intermodulation products occurs when the channel bit rate is greater than the laser linewidth. Crosstalk and maximum input power limitations for an FSK/DD system are calculated for up to 100 channels and channel spacing up to 100 GHz. It is shown that the spectral hole burning and dynamic carrier heating gain mechanisms must be included in addition to carrier modulation and gain saturation to accurately predict both incoherent and coherent crosstalk for channel spacing greater than 5 GHz. Degradation of the eye pattern due to coherent crosstalk is shown to exceed the incoherent contributions by up to 33 dB even when laser phase statistics and bit pattern statistics are accounted for. The result of this crosstalk mechanism is a predicted decrease in the maximum input power per channel to −43 dBm for 100 channels at 100 GHz channel spacing and −52 dBm for 100 channels at 5-GHz channel spacing for an optical SNR of 23 dB, representing a decrease in input power per channel of 25 dBm over incoherent crosstalk limitations alone.**

I. Introduction

SEMICONDUCTOR optical amplifiers (SOA's) show promise as compact amplifiers in frequency division multiplexed (FDM) optical transmission systems. Crosstalk induced by amplifier nonlinearities must be understood and minimized to make these systems practical. Previous models of multichannel crosstalk in SOA's assumed that primary and crosstalk fields are uncorrelated and accumulate incoherently [1]–[3]. However, in multigigabit systems with MHz linewidth lasers, the phase noise is negligible during the detection process [4] requiring that coherent interaction between fields be considered if the frequency difference between interacting fields falls within the receiver bandwidth. This coherent interaction is data dependent and can lead to BER floors and channel drop-out.

In this letter, we analyze the effect of coherent crosstalk, induced by FWM in SOA's on multichannel FSK direct detection (FSK/DD) systems. For multichannel amplification of FSK or PSK signals, FWM is the dominant crosstalk mechanism [3]. The two conditions required for coherent crosstalk are that the source linewidths are less than the bit rate and that the frequency difference of the interfering signals fall within the receiver bandwidth. For evenly spaced frequency systems, FWM fields coincide with primary channels. If the channel spacing is not exactly even, as may be the case with nonfrequency locked sources, the resulting beat frequencies that fall within the receiver bandwidth will have constant phase over the bit interval and coherent degradation will be detected. Prior work on multichannel crosstalk in SOA's considered only incoherent contributions from FWM and assumed that carrier modulation was the only process contributing to FWM gain, leading to significant crosstalk for channel spacings less than 1 GHz [3]. It has recently been shown that, in addition to carrier modulation, carrier heating and hole burning contribute significantly to FWM gain for frequency differences of up to several THz [5]. Therefore, these fast processes must be included in the crosstalk model to account for long range interaction in a multiple channel system.

The model presented here differs from previous models in that it accounts for coherent interaction among signal and FWM fields, bit statistics and phase statistics of the data and sources, and inclusion of hole burning, carrier heating, carrier modulation, and gain saturation in the FWM gain. We assume an N channel FSK/DD system with nominal channel frequency f_i and spacing $\Delta\nu$. A *space* is transmitted on frequency $f_i - \Delta f$ and a *mark* is transmitted on frequency $f_i + \Delta f$. After the SOA, the mark frequency of the center channel is passed through a narrowband optical filter and direct detected. Within the SOA, two-tone FWM mixing between fields at frequencies ω_i and ω_j generates fields at frequencies $2\omega_i - \omega_j$ and $2\omega_j - \omega_i$, and three-tone mixing between fields at frequencies ω_i, ω_j, and ω_k generates fields at frequencies $\omega_{ijk} = \omega_i + \omega_j - \omega_k$ over all possible combinations of i, j and k where $i \neq j \neq k$.

FWM in traveling wave SOA's involves scattering of an incident optical field off index and gain gratings created by beat frequency induced modulation of carriers and carrier occupation probabilities [6]. Fields A_j and A_k interfere in the SOA to produce a grating at frequency Ω_{jk} that scatters the field A_i into a new field A_{ijk}, such that for small $P_{\rm out}/P_s$ and large unsaturated gain,

$$A_{ijk} \approx -\eta_{jk} e^{3g_0 l/2} e^{-3P_{\rm out}/2P_s} A_i(0) A_j(0) A_k^*(0), \quad (1)$$

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The authors are with the Microelectronics Research Center (MiRC), School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332 USA.

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where l is the amplifier length and g_0 is the unsaturated amplifier gain per unit length. P_{out} and P_s are the SOA total output and saturation powers, respectively. The FWM coupling coefficient, η_{jk}, is approximated by a three term summation,

$$\eta_{jk} = \sum_{m=1}^{3} \frac{c_m}{(1 - i\Omega_{jk}\tau_m)}, \tag{2}$$

that is least-square fit to experimental data using the parameters c_m and τ_m [5]. The terms $m = 1, 2, 3$ represents the contribution of carrier modulation, carrier heating and hole burning, respectively. The unsaturated gain, gain saturation power and linewidth enhancement factor are included in c_m. The material response to the beat frequency $\Omega_{jk} = \omega_j - \omega_k$ results in gratings with relaxation time constants τ_m. In this letter, we assume a tensile-strained InGaAs–InGaAsP MQW traveling wave amplifier with the following experimentally determined constants: $\tau_1 = 200$ ps, $\tau_2 = 650$ fs, $\tau_3 = 50$ fs, $c_1 = 0.24\ e^{(-i1.30)}$, $c_2 = 0.0027e^{(i1.30)}$, and $c_3 = 0.00048\ e^{(i1.53)}$ [5]. For the calculations presented in this letter, the optical input is constrained to the unsaturated gain region. Therefore, the parameter c_m is independent of gain. Details on the amplifiers are given in references cited in [5].

The signal and FWM crosstalk power is found by coherently summing fields from the middle channel and all FWM generated 2- and 3-tone fields

$$P_{\text{tot}} = \left| A_{N/2} + \sum_{ijk} A_{ijk} \right|^2 = \left|A_{N/2}\right|^2 + \sum_{ijk} \left|A_{ijk}\right|^2 + \sum_{ijk}\sum_{i'j'k'} A_{ijk}A^*_{i'j'k'} + \left(\sum_{ijk} A_{N/2}A^*_{ijk} + \text{c.c.} \right). \tag{3}$$

Equation (3) identifies three crosstalk contributions: incoherent crosstalk from FWM fields (second term), beating between different FWM fields (third term), and beating between primary channels and FWM fields (fourth term). The subscripts i, j, and k are taken over all possible contributions between the N optical frequency channels that contribute to the mark frequency at $f_{N/2} + \Delta f = (f_i \pm \Delta f) + (f_j \pm \Delta f) - (f_k \pm \Delta f)$ and the primed subscripts are used to denote different combinations from unprimed subscripts. To describe degradation due to closing of the eye pattern, we define three power crosstalk terms associated with the second through fourth terms in (3)

$$C_{\text{I}} = \frac{\sum_{ijk} \left|A_{ijk}\right|^2}{\left|A_{N/2}\right|^2} \approx P^2(0)e^{-2P_{\text{out}}/P_s}e^{2g_0 l} \sum_{ijk} \left|\eta_{jk}\right|^2 \tag{4}$$

$$C_{\text{II}} = \frac{\sum_{ijk}\sum_{i'j'k'} A_{ijk}A^*_{i'j'k'}}{\left|A_{N/2}\right|^2} \approx P^2(0)e^{-2P_{\text{out}}/P_s}e^{2g_0 l} \times \left[\left| \sum_{ijk} \left|\eta_{jk}\right| e^{i(\theta_i+\theta_j-\theta_k+\theta_\eta)} \right|^2 - \sum_{ijk} \left|\eta_{jk}\right|^2 \right] \tag{5}$$

$$C_{\text{III}} = \frac{\left(\sum_{ijk} A_{N/2}A^*_{ijk} + \text{c.c.}\right)}{\left|A_{N/2}\right|^2} \approx -2P(0)e^{-P_{\text{out}}/P_s}e^{g_0 l} \times \text{Re}\left[e^{i\theta_{N/2}} \sum_{ijk} \left|\eta_{jk}\right| e^{-i(\theta_i+\theta_j-\theta_k+\theta_\eta)} \right], \tag{6}$$

where the factors, θ_i, θ_j, and θ_k are the phases of A_i, A_j and A_k respectively. The factor θ_η is the phase shift induced by η_{jk} and is different for each combination of $j, k, = 1 \cdots N$.

We define the crosstalk in terms of closing of the eye pattern. Assuming zero optical power in the mark channel when a space is transmitted, C_{I} closes the eye from the low state, C_{II} closes the eye from both the low and high states, and C_{III} closes the eye from the high state. The effect of bit statistics can be included by weighting each crosstalk term by the average probability of occurrence using coefficients defined in [7]. The total crosstalk, C_{IV}, is then given by

$$\begin{aligned} C_{\text{IV}} = {} & \frac{1}{4}C_{\text{I}}^{(2-\text{tone})} + \frac{3}{8}C_{\text{I}}^{(3-\text{tone},k\neq N/2)} + \frac{1}{4}C_{\text{I}}^{(3-\text{tone},k=N/2)} \\ & + \frac{1}{4}C_{\text{II}}^{(2-\text{tone})} + \frac{3}{8}C_{\text{II}}^{(3-\text{tone},k\neq N/2)} \\ & + \frac{1}{4}C_{\text{II}}^{(3-\text{tone},k=N/2)} \\ & + \sqrt{\frac{1}{2}}\sqrt{\frac{1}{4}}C_{\text{III}}^{(2-\text{tone})} + \sqrt{\frac{1}{2}}\sqrt{\frac{3}{8}}C_{\text{III}}^{(3-\text{tone},k\neq N/2)} \\ & + \sqrt{\frac{1}{2}}\sqrt{\frac{1}{4}}C_{\text{III}}^{(3-\text{tone},k=N/2)}. \end{aligned} \tag{7}$$

The effect of random variations of initial phases from bit-to-bit are accounted for by taking the ensemble average of the crosstalk with $\theta_i + \theta_j - \theta_k + \theta_\eta$ uniformly distributed over the interval $[0,\ 2\pi]$. We also consider the worst case crosstalk which is computed by setting $\theta_i + \theta_j - \theta_k + \theta_\eta = \pi$ for all FWM fields. Alignment of the polarization states and negligible dispersion in the SOA are assumed. The crosstalk for channel spacing $\Delta\nu = 5$ GHz is shown in Fig. 1 as a function of number of channels, N. The curve labeled $C_{\text{IV},A}$ is for the total phase averaged coherent crosstalk, $C_{\text{IV},W}$ is the total worst case, and C_{I} the incoherent only. In all cases, average bit statistics are included. The total crosstalk varies between the worst case and the incoherent case. However, it is important to note that error rate floors can appear when the crosstalk exceeds the incoherent level and can lead to channel drop-out in certain cases. The phase averaged coherent crosstalk exceeds the incoherent crosstalk by 23 dB for $N = 20$ and 35 dB for $N = 100$. In all curves, the total SOA output power is maintained at $P_{\text{out}}/P_s = 0.1$ to avoid the gain saturation region and accounts for the decrease in C_{I}. The unsaturated gain is $G_0 = 20$ dB and assumed to be independent of wavelength. The output saturation power $P_s^{\text{out}} = 14$ dBm. The inclusion of hole burning and carrier heating (solid lines) indicates 3 dB degradation for 100 channels at 5 GHz separation over that predicted by carrier modulation alone (dashed lines).

The maximum allowable input power per channel for an optical SNR of 23 dB is shown in Fig. 2 as a function of N with $\Delta\nu = 5$ GHz. The definitions of average and worst case are the same as described above. The coherent crosstalk

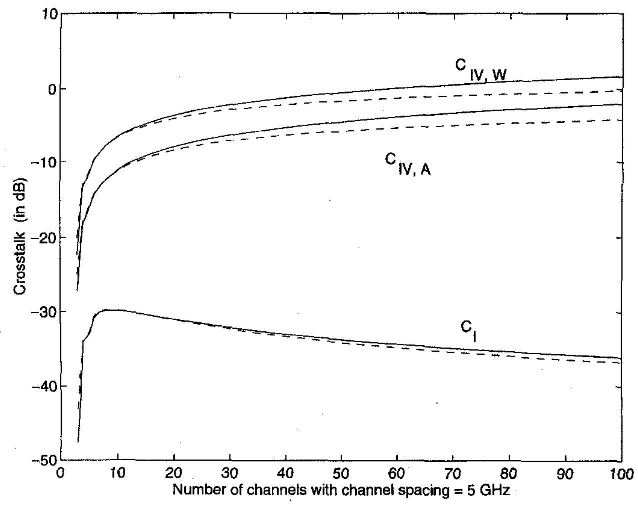

Fig. 1. FWM crosstalk contributions at middle channel mark frequency as a function of number of channels. C_I is incoherent, $C_{IV,A}$ is total coherent with averaging over laser phase and random bit arrivals, $C_{IV,W}$ is worst case coherent crosstalk with random bit arrivals. Inclusion of combined carrier modulation, hole burning, and carrier heating (solid lines) and carrier modulation only (dashed lines) are compared.

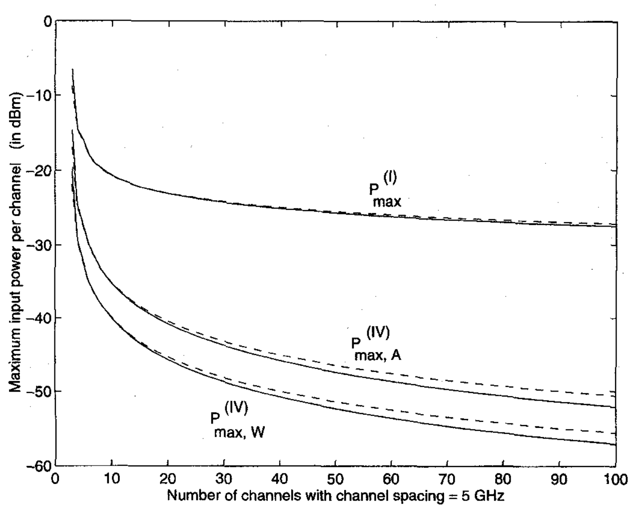

Fig. 2. Maximum input power per channel as a function of number of channels. $P_{\mathrm{max}}^{\mathrm{IV}}$ is the maximum power per channel due to coherent crosstalk and $P_{\mathrm{max}}^{\mathrm{I}}$ for incoherent crosstalk. Optical $\mathrm{SNR} = 23$ dB.

severely reduces the maximum allowable input power per channel over that from incoherent contributions alone by up to 24 dBm for the phase averaged case with $N = 100$, the per channel input power is limited to -52 dBm. The random phase case differs from worst case by only 5 dBm at 100 channels. The effect of hole burning and carrier heating are more pronounced as the channel spacing is increased out to 100 GHz as shown in Fig. 3, with $N = 100$ and $\mathrm{SNR} = 23$ dB. Both coherent and incoherent crosstalk are sensitive to inclusion of hole burning and carrier heating out beyond several GHz. Input power per channel is limited to -43 dBm at 100-GHz channel spacing for random phase and -48 dBm for the worst case. It is important to consider the worst case in system design even though the probability of its occurring

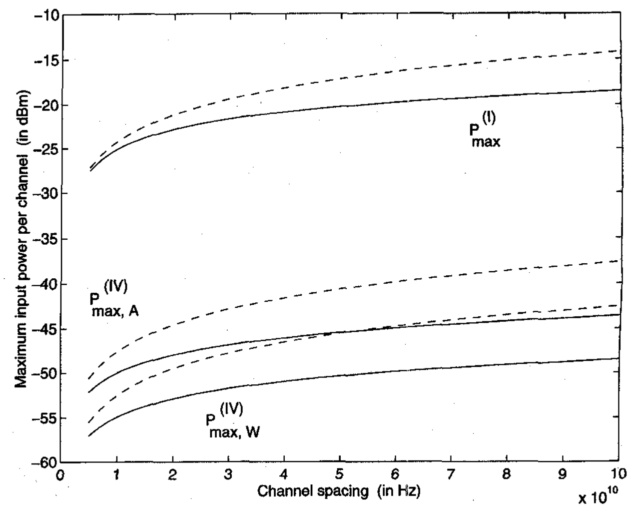

Fig. 3. Same as Fig. 2 as a function of channel spacing with $N = 100$.

is low since it can lead to momentary appearance of a severe bit error rate floor.

II. Summary

We have analyzed the influence of four-wave mixing in semiconductor optical amplifiers on coherently induced crosstalk in FDM FSK/DD transmission systems. The dominant crosstalk is due to beating between four-wave mixing generated fields and primary frequency channels that can severely degrade a direct detected system by limiting input power per channel by more than 25 dBm over limits due to incoherent crosstalk alone. Laser phase statistics and average bit pattern statistics were accounted for in this model. It was shown that spectral hole burning and dynamic carrier heating must be included in the gain model in addition to carrier modulation and gain saturation for channel spacings from several GHz out to 100 GHz. Calculations of crosstalk and maximum input power per channel were performed for up to 100 channels.

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